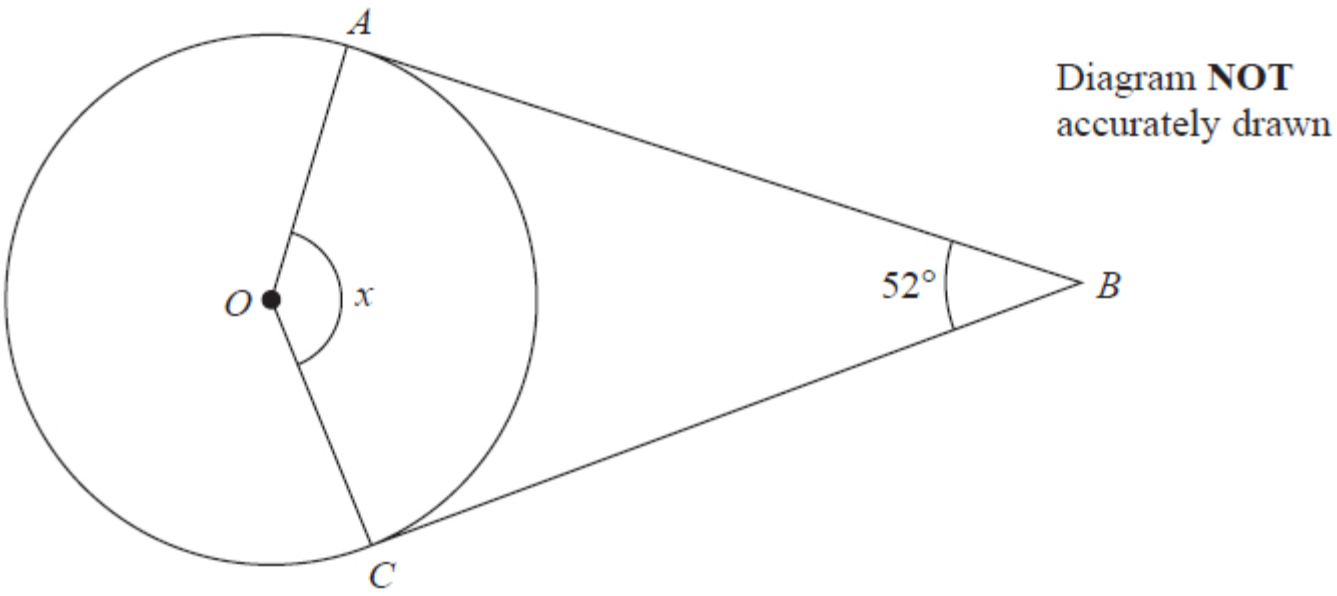
Unit 2 Revision Sheet E Circle Theorems Higher

Note: Higher tier students must also revise using the foundation tier revision worksheets as this content can also be assessed on higher tier papers.

Questions

Q1.



A and *C* are points on a circle, centre *O*.
AB and *CB* are tangents to the circle.
Angle *ABC* = 52°

Work out the size of angle *x*.
Give a reason for each stage of your working.

x = ... °

(Total for question = 4 marks)

Q2.

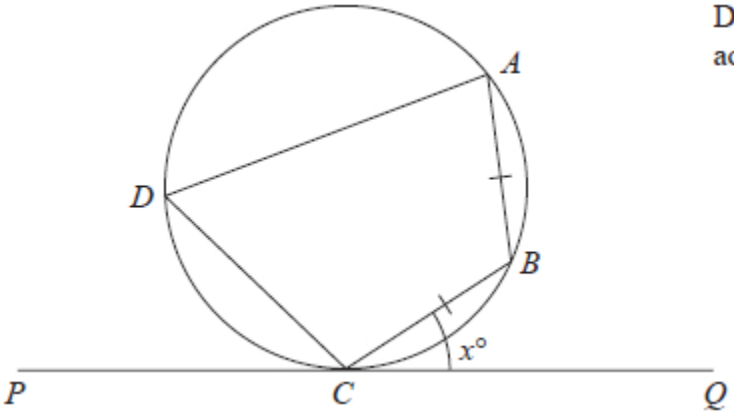


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle.
PCQ is a tangent to the circle.
AB = *CB*.

Angle *BCQ* = *x*°

Prove that angle *CDA* = 2*x*°
Give reasons for each stage in your working.

(Total for question = 5 marks)

Q3.

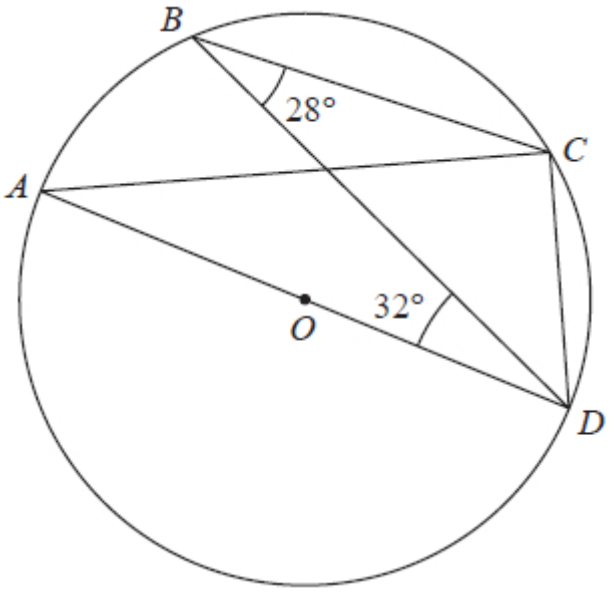


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*.
AOD is a diameter of the circle.

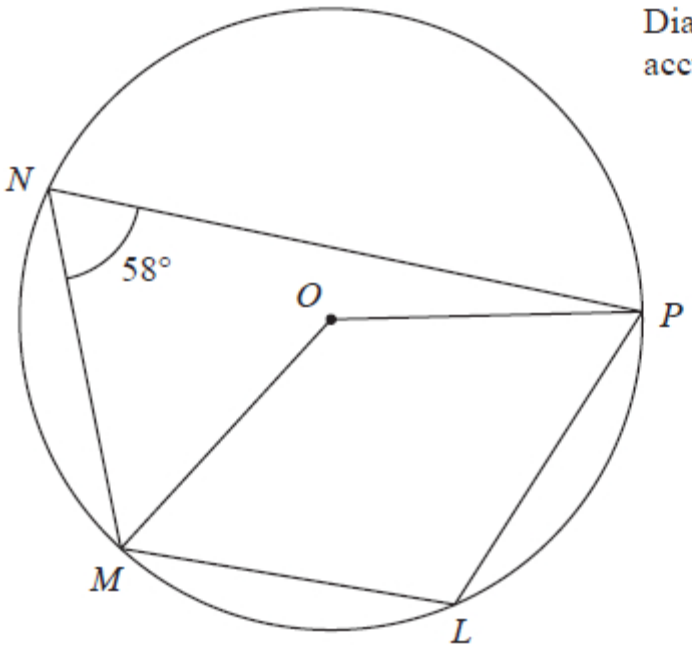
Angle *CBD* = 28°
Angle *BDA* = 32°

Find the size of angle *BDC*.
Give a reason for each stage of your working.

..... °

(Total for question = 4 marks)

Q4.



L, *M*, *N* and *P* are points on a circle, centre *O*

Angle *MNP* = 58°

(a) (i) Find the size of angle *MLP*

..... °

(ii) Give a reason for your answer.

.....

.....

(2)

(b) Find the size of the reflex angle *MOP*

..... °

(2)

(Total for question = 4 marks)

Q5.

P, *Q*, *R*, *S* and *T* are points on a circle with centre *O*.

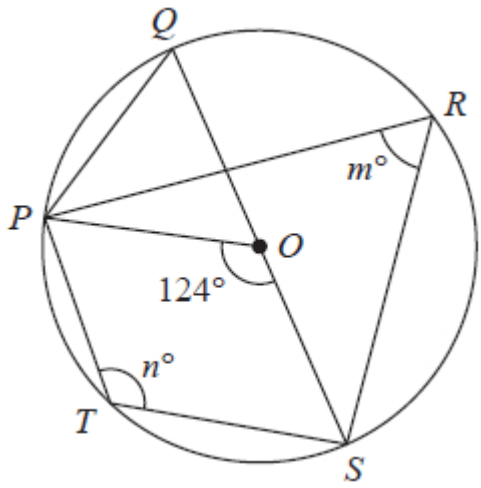


Diagram **NOT** accurately drawn

QOS is a diameter of the circle.

angle *POS* = 124° angle *PRS* = *m*° angle *PTS* = *n*°

(a) Find the value of

(i) *m*

.....

(ii) *n*

.....

(2)

(b) Find the size of angle *QPO*.

.....°

(1)

(Total for question = 3 marks)

Q6.

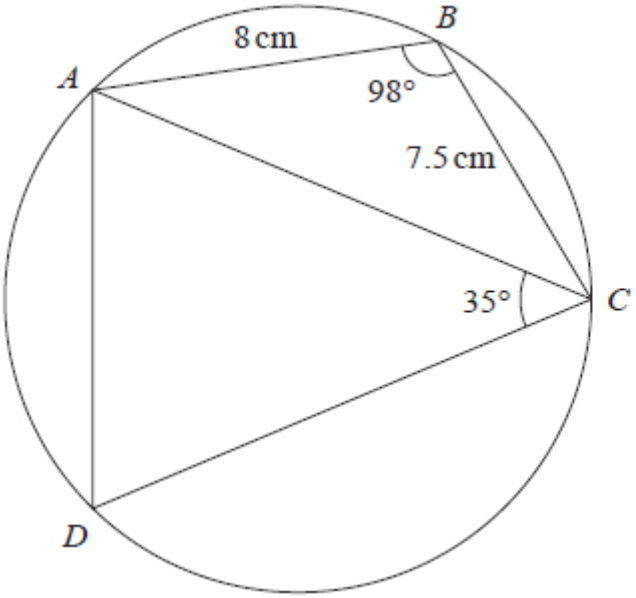


Diagram **NOT** accurately drawn

ABCD is a quadrilateral where *A*, *B*, *C* and *D* are points on a circle.

AB = 8 cm
BC = 7.5 cm
Angle *ABC* = 98°
Angle *ACD* = 35°

Work out the perimeter of quadrilateral *ABCD*.
Give your answer correct to one decimal place.

..... cm

(Total for question = 6 marks)

Q7.

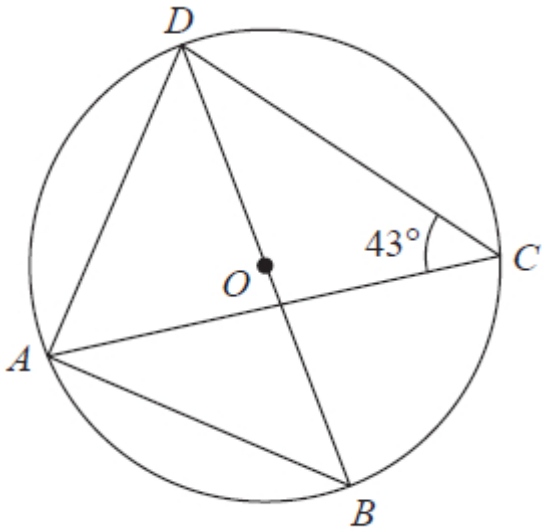


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*.
DOB is a diameter of the circle.
Angle *ACD* = 43°

Work out the size of angle *ADB*.
Give a reason for each stage in your working.

..... °

(Total for question = 5 marks)

Q8.

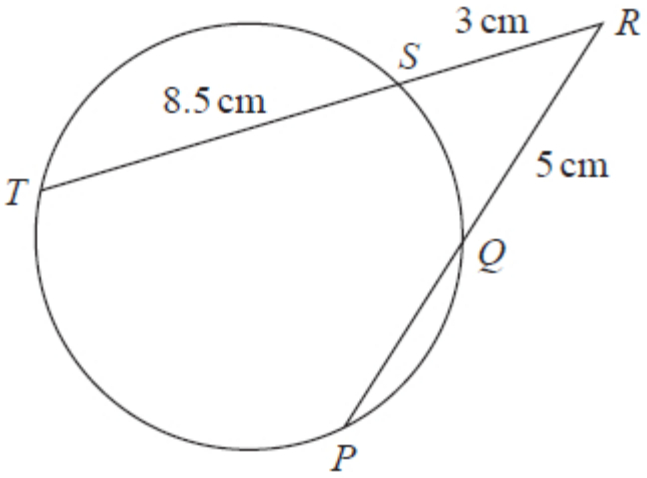


Diagram **NOT** accurately drawn

P, *Q*, *S* and *T* are points on a circle.
TSR and *PQR* are straight lines.

Work out the length of *PQ*.

..... cm

(Total for question = 3 marks)

Q9.

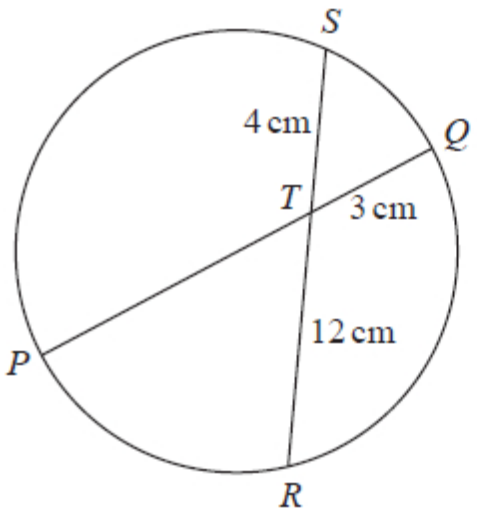


Diagram **NOT** accurately drawn

PTQ is a diameter of a circle.
RTS is a chord of the circle.

TQ = 3 cm ; *ST* = 4 cm ; *TR* = 12 cm

Calculate the radius of the circle.

..... cm

(Total for question = 3 marks)

Q10.

A, *B* and *C* are points on a circle, centre *O*

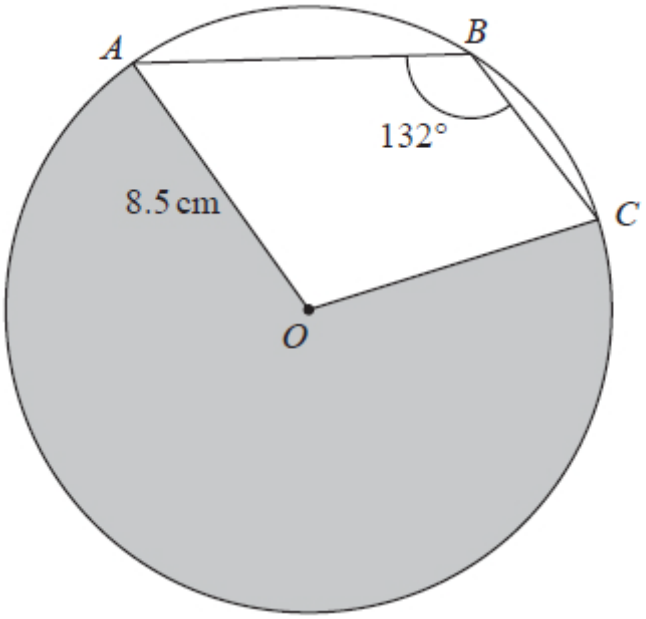


Diagram **NOT** accurately drawn

The radius of the circle is 8.5 cm
Angle *ABC* = 132°

Work out the perimeter of the shaded sector *AOC*
Give your answer correct to 3 significant figures.

..... cm

(Total for question = 3 marks)

Mark Scheme

Q1.

Q	Working	Answer	Mark	Notes
	$(x =)\ 360 - (90 + 90 + 52)$	128 Correct reasons	4	M1 A1 B1 The angle between a tangent and a radius is 90° oe B1 Angles in a quadrilateral add up to 360° oe
				Total 4 marks

Q2.

Question	Working	Answer	Mark	Notes	
	angle $CDB = x$ or angle $CAB = x$			M1	
	angle $CBA = 180 - 2x$			M1	
	angle $CDA = 180 - (180 - 2x) = 2x$			M1	
				B1	dep on M1 for any one appropriate circle theorem reason
		proof with reasons	5	A1	for complete proof with full reasons alternate segment theorem, angles in a triangle sum to 180°, isosceles triangle, opposite angles of a cyclic quadrilateral sum to 180°
	Alternative method				
	angle $CDB = x$ or angle $CAB = x$			M1	
	angle $ACB = x$			M1	
	angle $ACQ = 2x$ and angle $CDA = 2x$			M1	
				B1	dep on M1 for any one appropriate circle theorem reason
		proof with reasons	5	A1	for complete proof with full reasons alternate segment theorem, isosceles triangle

	Alternative method angle $OCB = 90 - x$ angle $BOC = 180 - 2(90 - x)$ $(=2x)$ angle $AOB = 2x$ and angle $CDA = 2x$			M1 M1 M1 B1 A1	 dep for any one appropriate circle theorem reason for complete proof with full reasons angle between tangent and radius is 90° oe, angles in a triangle sum to 180°, isosceles triangle, angle at centre is twice angle at circumference oe
		proof with reasons	5		
	eg angle ABC = $180 - 2x$ Angle CAB = angle ACB = $[180 - (180 - 2x)] \div 2 = x$ $BCQ = CAB = x$			M1 M1 M1 B1 A1	 Dep on M1 for any one appropriate circle theorem reason For complete proof with reasons e.g. opposite angles of cyclic quadrilateral sum to 180° angles in triangle sum to 180° isosceles triangle alternate segment theorem

Q3.

Question	Working	Answer	Mark	Notes
	Angle $CAD = 28°$ or angle $ACB = 32°$ or angle $ACD = 90°$ or angle $ABD = 90°$		4	M1
		30°		A1 For a correct answer of 30
	Angles in the **same segment** are equal, **angle** in a **semicircle** is 90° (or **angle** at centre is **double** angle at **circumference** oe) angles in a **triangle** add up to **180°/angles** in a **triangle** **isosceles** triangle **alternate** angles vertically **opposite angles** (or **vertically opposite)** **angles** at a **point** **opposite angles** in a **cyclic quadrilateral** angle between **tangent** and **radius (diameter)** **alternate segment** theorem **angles subtended** by the **same arc**(or **chord**) at the **circumference** (or **on the circle)**			B2 Dep on M1 for all correct reasons for their method used (if not B2 then award B1(dep on M1) for a correct circle theorem reason)
				Total 4 marks

Q4.

Q	Working	Answer	Mark	Notes
(a)(i)		122	1	B1
(a)(ii)		reason	1	B1 (dep on a correct answer or a correct method seen for (i)) Opposite angles in a cyclic quadrilateral sum to 180°
(b)	360 – 2 × 58 or 2 × '122'		2	M1 ft from (a)
		244		A1
				Total 4 marks

Q5.

Q	Working	Answer	Mark	Notes
(a) (i)		62	3	B1
(a) (ii)		118		B1ft 180 – their (a)(i)
(b)		62		B1
				Total 3 marks

Q6.

Q	Working	Answer	Mark	Notes
	$[ADC =]\ 180 - 98\ (= 82)$		6	M1 may be seen on diagram
	$[AC^2 =] 8^2 + 7.5^2 - 2\times 8\times 7.5\times\cos(98)\ (=136.95...)$			M1 correct equation for AC or AC^2
	$[AC =]\sqrt{"136.95"}$ **or** $\sqrt{64 + 56.25 + 16.7....}\ (=11.7...)$ oe			M1 complete method to find AC showing correct order of operations
	eg $[[AD =]\dfrac{"11.7"\sin 35}{\sin "82"}\ (= 6.77..)$ **or** $[DC =]\dfrac{"11.7"\times\sin "63"}{\sin "82"}\ (=10.5..)$oe (where "**82**" = **180 – 98** , "63" = 180 – "82" – 35)			M1 correct calculation for AD **or** DC dep on 1st M1 and 2nd M1
	eg $[AD =]\dfrac{"11.7"\sin 35}{\sin "82"}$ **and** $[DC =]\dfrac{"11.7"\sin "63"}{\sin "82"}$ oe **or** $[AD =]\dfrac{"11.7"\sin 35}{\sin "82"}$ **and** $[DC =]\sqrt{"11.7"^2 + "6.77"^2 - 2\times "11.7"\times "6.77"\times\cos "63"}$ $[DC =]\dfrac{"11.7"\sin "63"}{\sin "82"}$ **and** $[AD =]\sqrt{"11.7"^2 + "10.5"^2 - 2\times "11.7"\times "10.5"\times\cos 35}$ Where "63" = 180 – "82" – 35			M1 correct calculations for AD **and** DC ($AD = 6.77...$ $DC = 10.5...$) dep on 1st M1 and 2nd M1
	Working not required, so correct answer scores full marks (unless from obvious incorrect working)	32.8		A1 accept 32.7 – 32.9
				Total 6 marks

Q7.

Q	Working	Answer	Mark	Notes	
	Angle $DBA = 43°$ **or** Angle $DAB = 90°$			M1	
	$180 - 90 - 43$ (=47)			M1	
				A1	
		47 with reasons	5	B2	for full reasons Angles in the same segment are equal; Angle in a semi-circle is a right angle Angles in a triangle add up to 180° (B1 for a correct and relevant reason using a circle theorem)
					Total 5 marks

Q8.

Q	Working	Answer	Mark	Notes	
	$3 \times (3 + 8.5) = 5 \times PR$ **or** $3 \times (3 + 8.5) = 5 \times (5 + PQ)$			M1	
	$(3 \times (3 + 8.5)) \div 5 \ - 5$			M1	for a complete method for PQ
		1.9	3	A1	
					Total 3 marks

Q9.

Q	Working	Answer	Mark	Notes
	$(PT=)\ \frac{12\times4}{3}\ (=16)$		3	M1 NB: 16 from 12 + 4 is incorrect working
	$(r=)$ ("16" + 3) ÷ 2			M1
		9.5		A1 oe
				Total 3 marks

Q10.

Q	Working	Answer	Mark	Notes
	$(\angle AOC=)132\times2(=264)$		3	M1 for method to find angle at the centre. Do not award this mark if contradicted on the diagram eg if obtuse AOC is labelled as 264
	eg $\frac{"264"}{360}\times2\times\pi\times8.5\ (=39.1...$ or $\frac{187}{15}\pi)$ or $2\times\pi\times8.5-\frac{360-"264"}{360}\times2\times\pi\times8.5\ (=39.1...$ or $\frac{187}{15}\pi)$ or $\frac{"264"}{360}\times2\times\pi\times8.5+2\times8.5$ or $2\times\pi\times8.5-\frac{360-"264"}{360}\times2\times\pi\times8.5+2\times8.5$			M1 for a method to find the length of arc AC or perimeter of the sector – allow use of their AOC as long as clearly labelled
	Correct answer scores full marks (unless from obvious incorrect working)	56.2		A1 accept 56.1 – 56.2
				Total 3 marks